## GCE AS/A level

0976/01

# MATHEMATICS - C4 <br> Pure Mathematics 

A.M. FRIDAY, 12 June 2015

1 hour 30 minutes

## ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.


## INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.
Answer all questions.
Sufficient working must be shown to demonstrate the mathematical method employed.

## INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.
You are reminded of the necessity for good English and orderly presentation in your answers.

1. Given that $f(x)=\frac{2 x^{2}+5 x+25}{(x+3)^{2}(x-1)}$,
(a) express $f(x)$ in terms of partial fractions,
(b) evaluate

$$
\int_{3}^{10} f(x) \mathrm{d} x
$$

giving your answer correct to two decimal places.
2. The curve $C$ has equation

$$
x^{4}+3 x^{2} y-2 y^{2}=34
$$

(a) Show that $\frac{\mathrm{d} y}{\mathrm{~d} x}=\frac{4 x^{3}+6 x y}{4 y-3 x^{2}}$.
(b) Find the coordinates of each of the points on $C$ where the tangent is parallel to the $y$-axis.
3. (a) Find all values of $x$ in the range $0^{\circ} \leqslant x \leqslant 180^{\circ}$ satisfying

$$
\begin{equation*}
\tan \left(x+45^{\circ}\right)=8 \tan x \tag{5}
\end{equation*}
$$

(b) (i) Express $\sqrt{13} \sin \theta-6 \cos \theta$ in the form $R \sin (\theta-\alpha)$, where $R$ and $\alpha$ are constants with $R>0$ and $0^{\circ}<\alpha<90^{\circ}$.
(ii) Find all values of $\theta$ in the range $0^{\circ} \leqslant \theta \leqslant 360^{\circ}$ satisfying

$$
\begin{equation*}
\sqrt{13} \sin \theta-6 \cos \theta=-4 \tag{6}
\end{equation*}
$$

4. The line $L$ has equation $y=m x$, where $m>0$. The region $R$ is bounded by $L$, the $x$-axis and the line $x=a$, where $a>0$.
(a) Using integration, find an expression, in terms of $a$ and $m$, for the volume $V$ generated when $R$ is rotated through four right angles about the $x$-axis.
(b) The point with coordinates $(a, b)$ lies on $L$.
(i) Rewrite the expression for the volume $V$ found in part (a) in terms of $a$ and $b$.
(ii) Give a geometrical interpretation of your answer.
5. Expand $\left(1+\frac{x}{8}\right)^{-\frac{1}{2}}$ in ascending powers of $x$ up to and including the term in $x^{2}$.

State the range of values of $x$ for which your expansion is valid.
Hence, by writing $x=1$ in your expansion, find an approximate value for $\sqrt{2}$ in the form $\frac{a}{b}$,
where $a$ and $b$ are integers whose values are to be found.
6. The parametric equations of the curve $C$ are $x=a t^{2}, y=2 a t$, where $a$ is a positive constant. The points $P$ and $Q$ lie on $C$ and have parameters $p$ and $q$ respectively.
(a) Simplifying your answer in each case, find
(i) the gradient of the tangent to $C$ at the point $P$,
(ii) the equation of the tangent to $C$ at the point $P$.
(b) (i) Find an expression, in its simplest form, for the gradient of the line $P Q$.
(ii) Explain how you could use the answer of (b)(i) to derive the gradient of the tangent to $C$ at the point $P$.
7. (a) Use the substitution $u=12-x^{3}$ to evaluate

$$
\begin{equation*}
\int_{0}^{2} \frac{x^{2}}{\left(12-x^{3}\right)^{2}} d x \tag{4}
\end{equation*}
$$

(b) (i) Find $\int x \cos 2 x \mathrm{~d} x$.
(ii) Use the result of (b)(i) to find

$$
\begin{equation*}
\int x \sin ^{2} x \mathrm{~d} x \tag{7}
\end{equation*}
$$

8. The position vectors of the points $A$ and $B$ are given by

$$
\begin{aligned}
& \mathbf{a}=5 \mathbf{i}-\mathbf{j}-\mathbf{k}, \\
& \mathbf{b}=4 \mathbf{i}-3 \mathbf{j}+6 \mathbf{k},
\end{aligned}
$$

respectively.
(a) (i) Write down the vector $\mathbf{A B}$.
(ii) Find the vector equation of the line $A B$.
(b) The vector equation of the line $L$ is given by

$$
\mathbf{r}=2 \mathbf{i}-3 \mathbf{j}-4 \mathbf{k}+\mu(\mathbf{i}+\mathbf{j}-\mathbf{k})
$$

Show that the lines $A B$ and $L$ intersect and find the position vector of the point of intersection.
9. A bookseller values a rare book at $£ A$ on August 1 st 2010. The value, $£ P$, of the book $t$ years after this date may be modelled as a continuous variable. The rate of increase of $P$ may be assumed to be directly proportional to $P^{2}$.
(a) Write down a differential equation satisfied by $P$.
(b) Show that

$$
\frac{1}{k}\left(\frac{P-A}{P A}\right)=t
$$

where $k$ is a constant.
(c) The value of the book is $£ 800$ on August 1st 2013 and $£ 900$ on August 1st 2014 . Find the value of $A$.
10. Prove by contradiction the following proposition.

If $a$ and $b$ are odd integers such that 4 is a factor of $a-b$, then 4 is not a factor of $a+b$.

The first lines of the proof are given below.
Assume that 4 is a factor of $a+b$.
Then there exists an integer $c$ such that $a+b=4 c$.

END OF PAPER

