

**GCE AS/A level** 

0976/01



S15-0976-01

A.M. FRIDAY, 12 June 2015

1 hour 30 minutes

## ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

## INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen. Answer **all** questions. Sufficient working must be shown to demonstrate the **mathematical** method employed.

## INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question. You are reminded of the necessity for good English and orderly presentation in your answers.

- Given that  $f(x) = \frac{2x^2 + 5x + 25}{(x+3)^2(x-1)}$ , 1.
  - (a) express f(x) in terms of partial fractions,
  - (b) evaluate

$$\int_{3}^{10} f(x) \,\mathrm{d}x,$$

giving your answer correct to two decimal places. [3]

2. The curve C has equation

$$x^4 + 3x^2y - 2y^2 = 34.$$

(a) Show that 
$$\frac{dy}{dx} = \frac{4x^3 + 6xy}{4y - 3x^2}$$
. [3]

- Find the coordinates of each of the points on C where the tangent is parallel to (b) the *v*-axis.
- Find all values of x in the range  $0^{\circ} \leq x \leq 180^{\circ}$  satisfying 3. (a)

$$\tan(x+45^{\circ}) = 8\tan x.$$
 [5]

- Express  $\sqrt{13}\sin\theta 6\cos\theta$  in the form  $R\sin(\theta \alpha)$ , (b) (i) where *R* and  $\alpha$  are constants with R > 0 and  $0^{\circ} < \alpha < 90^{\circ}$ .
  - Find all values of  $\theta$  in the range  $0^{\circ} \leq \theta \leq 360^{\circ}$  satisfying (ii)

$$\sqrt{13}\sin\theta - 6\cos\theta = -4.$$
 [6]

- The line *L* has equation y = mx, where m > 0. The region *R* is bounded by *L*, the *x*-axis and the 4. line x = a, where a > 0.
  - Using integration, find an expression, in terms of *a* and *m*, for the volume *V* generated (a) when R is rotated through four right angles about the x-axis. [3]
  - The point with coordinates (a, b) lies on L. (b)
    - Rewrite the expression for the volume V found in part (a) in terms of a and b. (i)
    - Give a geometrical interpretation of your answer. (ii) [3]

[4]

[4]

Expand  $\left(1+\frac{x}{8}\right)^{-\frac{1}{2}}$  in ascending powers of x up to and including the term in  $x^2$ . 5.

State the range of values of *x* for which your expansion is valid. Hence, by writing x = 1 in your expansion, find an approximate value for  $\sqrt{2}$  in the form  $\frac{a}{b}$ , where *a* and *b* are integers whose values are to be found. [5]

- The parametric equations of the curve C are  $x = at^2$ , y = 2at, where a is a positive constant. The 6. points *P* and *Q* lie on *C* and have parameters *p* and *q* respectively.
  - (a) Simplifying your answer in each case, find
    - the gradient of the tangent to C at the point P, (i)
    - (ii) the equation of the tangent to C at the point P.
  - (b) Find an expression, in its simplest form, for the gradient of the line PQ. (i)
    - Explain how you could use the answer of (b)(i) to derive the gradient of the tangent (ii) to C at the point P. [4]
- 7. Use the substitution  $u = 12 - x^3$  to evaluate (a)

$$\int_{0}^{2} \frac{x^{2}}{\left(12 - x^{3}\right)^{2}} \, \mathrm{d}x \,. \tag{4}$$

- (i) Find  $x \cos 2x \, dx$ . (b)
  - Use the result of (b)(i) to find (ii)

$$x\sin^2 x \,\mathrm{d}x.$$
 [7]

The position vectors of the points A and B are given by 8.

$$a = 5i - j - k,$$
  
 $b = 4i - 3j + 6k,$ 

respectively.

- (a) (i) Write down the vector **AB**.
  - (ii) Find the vector equation of the line *AB*.
- (b) The vector equation of the line L is given by

$$\mathbf{r} = 2\mathbf{i} - 3\mathbf{j} - 4\mathbf{k} + \boldsymbol{\mu}(\mathbf{i} + \mathbf{j} - \mathbf{k}).$$

Show that the lines AB and L intersect and find the position vector of the point of intersection. [6]



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[3]

[4]

- **9.** A bookseller values a rare book at  $\pounds A$  on August 1st 2010. The value,  $\pounds P$ , of the book *t* years after this date may be modelled as a continuous variable. The rate of increase of *P* may be assumed to be directly proportional to  $P^2$ .
  - (a) Write down a differential equation satisfied by *P*. [1]
  - (b) Show that

$$\frac{1}{k} \left( \frac{P-A}{PA} \right) = t,$$

where k is a constant.

- (c) The value of the book is £800 on August 1st 2013 and £900 on August 1st 2014. Find the value of *A*.
- **10.** Prove by contradiction the following proposition.

If *a* and *b* are odd integers such that 4 is a factor of a - b, then 4 is **not** a factor of a + b.

The first lines of the proof are given below.

Assume that 4 is a factor of a + b. Then there exists an integer *c* such that a + b = 4c. [3]

## **END OF PAPER**

[4]

[3]